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NASA Program Apollo Working Paper No. 1111

THE PREDICTION OF INFLIGHT DYNAMIC

STRESSES IN TYPICAL SPACECRAFT SHELL STRUCTURES

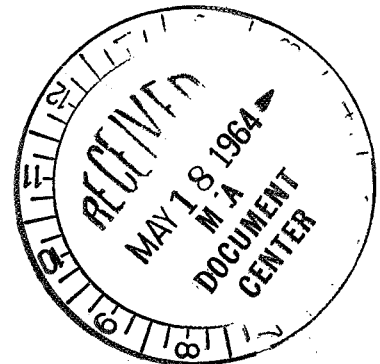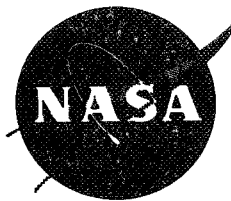N70-35657

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MANNED SPACECRAFT CENTER

Houston, Texas

April 3, 1964

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THE PREDICTION OF INFLIGHT DYNAMIC

STRESSES IN TYPICAL SPACECRAFT SHELL STRUCTURES

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THE PREDICTION OF INFLIGHT DYNAMIC
STRESSES IN TYPICAL SPACECRAFT SHELL STRUCTURES

SUMMARY

A method is presented for determining the inflight dynamic stresses (rms) in cylindrical shells using experimental data from ground vibration tests to establish the natural modes and frequencies, and wind tunnel or flight test measurements to establish the magnitude and frequency of the fluctuating pressure environment. This method was used for predicting stresses in a section of the Apollo boilerplate structure, and a portion of these results are shown as an example of the method.

INTRODUCTION

For a limited period during boost flight when velocities are in the transonic range, most spacecraft configurations are subjected to an intense fluctuating pressure environment. The dynamic stresses produced by the response of the vehicle structure to these fluctuating pressures must be analyzed and combined with the static stresses that result from inertia and air loads in order to evaluate the structural integrity of the spacecraft.

Assuming that scaling laws are sufficiently well known, the magnitude and frequency spectrum of the external fluctuating pressures may be obtained from wind tunnel experiments using scaled models; and the vibration characteristics of the structure may be obtained by non-destructive testing of a full scale prototype structure. Based on the results of such experiments, a method of predicting the inflight dynamic stresses (rms) is presented in this paper, together with an example of the predicted response of a portion of the Apollo boilerplate structure. There is at present no way of knowing the exact spatial correlation factor between applied force and modal deflection and an arbitrary factor of 0.2 has therefore been assumed in the example. No attempt is made in this paper to determine the probability of exceeding stresses that are higher than the rms values.

SYMBOLS

A	Maximum deflection - inches
C_f	Maximum distance from neutral axis to outside fiber - inches
E	Modulus of elasticity - psi
F_n	Generalized force - lbs per cps
f_n	Natural frequency - cps
M_n	Generalized mass - slugs
R	Radius - inches
S	Area - ft^2
S_n	Power spectrum - lbs^2 per radians/sec
U	Radial deflection - feet
W	Weight - lbs
a	Acceleration - ft per sec^2
g	Earth gravitational acceleration - 32.2 ft per sec^2 (386 in per sec^2)
n	Number of half waves
p_{rms}	Root mean square pressure - lbs per ft^2
s	Peripherial length - inches
γ	Ratio of damping coefficient to critical damping coefficient
ω_n	Natural frequency - radians per sec
σ	Bending stress in circumferential direction - psi
α	Angle of attack

METHOD OF ANALYSIS

General

The method presented is a classical modal solution for the response of a damped vibrating system to a sinusoidal forcing function, and is based on the premise that for extremely low values of structural damping, there is no "coupling" between the various natural modes. The contribution of all forces except those at or near the particular resonant frequency being considered is therefore negligible.

The natural modes and frequencies of the structure together with the ratio of structural damping coefficient to critical damping coefficient are obtained experimentally from a ground vibration test of the structure. The methods used for conducting these tests on an Apollo boilerplate service module structure is explained in reference 1 which is to be published in the very near future.

The accuracy and quantity of measurements taken during the vibration test cannot be over-emphasized since the final answer can be no more accurate than the original input data, and too few or inaccurate measurements may result in erroneous conclusions.

Normalized Ring Stresses

The normalized ring stresses are defined here as the stresses obtained as a direct result of the vibration test using an arbitrary shaker force input. In order to facilitate the numerical computations, the procedure is to divide the shell into a grid-work of circumferential bands and longitudinal strips. The size of the grid is arbitrary and should be determined by the accuracy desired since the use of a larger number of grid elements will result in greater accuracy.

From the ground vibration test values of radial acceleration, contour plots of lines of constant acceleration (a/g) over the entire shell for each natural frequency are plotted as shown in figure 1, and the areas of the shell experiencing high accelerations are selected for stress analysis. If the shell under consideration is non-uniform, for example, having rings of variable moments of inertia, the areas under highest accelerations may not produce the highest stress, and it may be necessary to investigate bands adjacent to the one having peak accelerations.

By plotting values of (a/g) for each selected circumferential band and for each selected natural frequency and drawing a smooth curve (approximately sinusoidal) through the points, as shown in figure 2, the

deflected shape of the particular band when vibrating at this discrete frequency is defined.

Since the deflected shape of the shell band is assumed to be approximated by a sine wave, it follows that:

$$U = A \sin n\theta = A \sin \frac{ns}{R}$$

$$\therefore \frac{d^2 U}{ds^2} = -\frac{An^2}{R^2} \sin \frac{ns}{R} = -\frac{Un^2}{R^2}$$

And since for small deflections

$$\frac{d^2 U}{ds^2} = -\frac{M}{EI} = -\frac{\sigma}{EC_f}$$

$$\therefore \sigma = EC_f U \frac{n^2}{R^2} \tag{1}$$

It should be kept in mind that the above ring stress is derived from rms accelerations resulting from the structure being vibrated at a particular frequency using an arbitrary shaker force and as such represents only a "unit" rms stress which must be multiplied by a "response" magnification factor to obtain an inflight rms stress.

Response of Structure

The method of determining the structural response during flight is derived from reference 2 and only the method of obtaining the parameters for the response equations and assumptions used will be given here.

Terms necessary to determine structural response are the generalized mass, generalized force, damping coefficient, power spectral density and frequency.

Generalized Mass. - The generalized mass, M_n, is defined as the effective accelerated mass of a structure for a particular mode. It is obtained when a multi-degree of freedom system is treated as a single degree system, by virtue of the fact that motion in a particular natural mode does not produce forces in any of the other modes.

To obtain the generalized mass, the weight of the structure and ballast or other weight in each element of the previously mentioned grid is computed and the weight assumed to be concentrated at the center of the element. From the contour plots of normalized accelerations (a/g) for each natural frequency, measured acceleration values are assigned to the corresponding grid elements, and the sum of the product of mass and normalized displacement squared over the entire surface of the shell then gives the generalized mass, that is:

$$M_n = \frac{\Sigma W \ (a/g)^2}{g} \ \text{Slugs}$$ (2)

Generalized Force.- Generalized force, F_n, is defined as the effective force on the mode produced by all external forces on the structure when factored by the normalized mode displacement.

This force is caused by the fluctuating pressure occurring along the length of the structure and is determined experimentally by means of transducers located along the vehicle during either flight or wind tunnel tests. Measured fluctuating pressure is in terms of sound pressure level (S.P.L.) for a particular reference pressure (usually 0.0002 dynes per square centimeter) and when plotted versus station the S.P.L. for each band can be determined. Figure 3 illustrates the one-third octave S.P.L. obtained from wind tunnel tests of the Apollo configuration.

Sound pressure levels are usually measured as the sound pressure level between two frequencies $(f_2 - f_1)$ with the width of the pass band depending upon the equipment used. For this application the sound pressure level for a band of 1 cps width, that is spectrum level, is desired. Conversion from the one-third octave bandwidth sound pressure level to the spectrum level (1 cps band level) is accomplished by the following equation taken from reference 3.

$$\text{db/cps} = \frac{\text{db}}{\frac{1}{3} \ \text{octave}} - 10 \log \frac{\Delta f}{1}$$ (3)

where Δf is the $\frac{1}{3}$ octave bandwidth.

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The spectrum level S.P.L. is now changed to rms pressure by the equation:

$$S.P.L. = 20 \log 10 \left(\frac{P}{P_{ref}} \right) \tag{4}$$

where

$$P_{ref} = 29.1 \times 10^{-10} \text{ psi} = 0.0002 \text{ Dynes/cm}^2$$

Assuming there is 100 percent correlation between the direction of the applied forces and the modal displacement, the rms amplitude spectrum of the generalized force at any natural frequency is defined as:

$$F_n(\omega_n) = \sum_{\text{All elements}} \frac{P_{rms}}{cps} \left(\frac{a}{g} \right) \Delta S \tag{5}$$

where $F_n(\omega_n)$ = generalized fo.ce (rms) - lbs per cps

P = pressure - psf

a/g = normalized radial acceleration

ΔS = surface area of element. - ft^2

The power spectrum of the generalized force which is required for the response calculation is then obtained by squaring the amplitude spectrum given in equation 5.

Since the amplitude spectrum of the pressure $\left(\frac{P_{rms}}{cps} \right)$ is assumed uniform over the circumference and if all elements of area (ΔS) in the band are equal, it is only necessary to sum the absolute values of the modal displacement $\left(\Sigma \left| \frac{a}{g} \right| \right)$ in the band and factor by $S \left| \frac{P_{rms}}{cps} \right|$, where S represents the area of one band.

The contribution to the generalized force per band is then:

$$\Delta F_n = S \left| \frac{Prms}{cps} \right| \left(\sum \left| \frac{a}{g} \right| \right) \tag{6}$$

and the value of the rms generalized force for the whole structure is then:

$$F_n(\omega_n) = \sum_{Band\ z}^{Band\ a} S \left| \frac{Prms}{cps} \right| \left(\sum \left| \frac{a}{g} \right| \right) \text{ lbs per cps} \tag{7}$$

Response Ratio.- The mean square displacement of the normalized, generalized coordinate assuming a linear response is found by the equation derived in reference 2.

$$\overline{U}^2 = \frac{\pi\ S_n}{4\ \gamma\ M_n^2\ \omega_n^3} \tag{8}$$

and

$$\therefore \text{RMS } (\overline{U}^2)^{\frac{1}{2}} = \left(\frac{\pi}{4\ \gamma} \right)^{\frac{1}{2}} \frac{S_n^{\frac{1}{2}}}{M_n\ \omega_n^{\frac{3}{2}}} \tag{9}$$

where

$$S_n = \frac{F_n^2}{2\ \pi} \text{ lbs}^2/\text{radians per second}$$

Since the motion being considered is harmonic, the relationship between acceleration and deflection is:

$$\ddot{U}_{rms} = \overline{U}\ \omega_n^2 \tag{10}$$

and the rms acceleration is therefore,

$$\left(\overline{U^2}\right)^{\frac{1}{2}} = \left(\frac{\omega_n}{8\gamma}\right)^{\frac{1}{2}} \frac{F_n}{M_n} \tag{11}$$

$$\therefore \frac{U}{g}\bigg|_{rms} = \frac{1}{g}\left(\frac{\omega_n}{8\gamma}\right)^{\frac{1}{2}} \frac{F_n}{M_n} \tag{12}$$

The damping values represented by γ in the above equations are obtained from the ground vibration tests and are assumed to remain constant with amplitude of deflection.

To obtain the rms dynamic stresses, for 100 percent correlation the stresses obtained by equation 1 must be multiplied by the above amplification factor since the radial deflection used in equation 1 is for acceleration values (a/g) obtained directly from the vibration test using an arbitrary shaker force.

Factors to account for less than 100 percent spatial correlation are discussed in the example problem.

Example Problem

For illustration purposes a recent test of the Apollo boilerplate service module, insert, and adapter structure and calculations of resulting fluctuating stresses at Mach number of 0.8 will be used. This structure is identical with the boilerplate spacecraft that will be flown on the Saturn Research and Development mission and the external dimensions are shown in figure 4.

Structural Description.- The types of construction used for the three components investigated are similar. The semi-monocoque structure of the service module consists of an aluminum skin which is reinforced with rings and longerons. The insert and adapter have stringers in addition to the longerons. The axial load in the structure is carried by six equally spaced, heavy longerons which are riveted to the skin and which extend the entire length of each section of the boilerplate. The longerons are made up of two steel tees joined by an aluminum web and are of constant depth in the insert and adapter as shown in figure 5. The depth of the longerons in the service module varies linearly from a maximum of 17.2 inches at the top end where it mates with the command module to 5.50 inches at the bottom end. The structural details of the longerons are shown in figures 5 and 6. The skin of all three components is made from 2024-T3 aluminum alloy and is of a constant thickness of 0.16 inches over the entire boilerplate. The skin is reinforced with a

total of thirty rings. The rings in the three components are all made from 2024-T4 aluminum with the exception of the bottom ring of the adapter which is made from steel. The rings in the service module are evenly spaced at intervals of 12.4 inches wereas the rings in the insert and adapter are spaced at unequal intervals. The cross-sectional dimensions of typical rings in the service module, insert and adapter are shown in figure 7. In addition to the six heavy longerons, there are a total of 28 tee-shaped stringers made from 7075-T6 aluminum in the insert and adapter.

The structural weight of the test boilerplate is 6,752 lbs. In order to simulate the weight and center of gravity of the production Apollo service module, a total of 3,082 lbs of lead ballast is attached to the service module longerons as illustrated in figures 8 and 9. The ballast weight is in addition to the total structural weight of 6,752 lbs.

The surface area of the shell is divided into a grid work of 26 (designated A to Z) horizontal circular bands and 36 vertical strips of equal arc width as shown in figure 8.

Procedure.- By vibrating the structure at its natural frequencies using an arbitrary shaker force input, the normalized accelerations (a/g) over the entire shell are obtained and plotted as contour lines of constant acceleration. A typical plot for a frequency of 89.8 cps is shown in figure 1.

Band N at station 171.4 is selected for analysis at this natural frequency since it appears from figure 1 to be subjected to the highest accelerations. Values of (a/g) are then plotted around band N as shown in figure 2. From figure 2 a maximum a/g of 4.75 and a corresponding wave length of n = 3.0 is obtained. Parameters of the ring (including 12.4 inches of shell skin) are:

$$E = 10.3 \times 10^6 \text{ psi} \qquad C_f = 5.7 \text{ inches} \qquad R = 75.54 \text{ inches}$$

The maximum rms radial deflection and bending stress in the ring flange under the vibration forces imposed during the shake test may now be obtained as shown below from the expression of equation 1.

$$U_{max} = g \frac{(a/g)}{\omega_n^2} = 386 \frac{4.75}{(2 \pi 89.8)^2}$$

$$= 5.74 \times 10^{-3} \text{ ins.}$$

and

$$\sigma = EC_f U \left(\frac{n}{R}\right)^2 = 10.3 \times 10^6 \times 5.7 \times 5.74 \times 10^{-3} \times \left(\frac{3}{75.54}\right)^2$$

$$= 534 \text{ psi (rms)}$$

The above stress may be regarded as a "unit" stress which must be multiplied by an amplification factor (dependent on the ratio of generalized force to generalized mass) in order to obtain the fluctuating rms stresses during flight.

In order to calculate the generalized mass, the weights of the structure and ballast are computed from detail drawings and the weights enclosed in each square of the grid are assigned to the center of each element of the grid shown in figure 8. The generalized mass is then calculated in tabular form as shown in table I from the expression given in equation 2. In this example, the generalized mass, M_n, is 571 slugs when $n = 3$ and $f_n = 89.8$ cps.

To determine the generalized force, wind tunnel data from a test conducted on a 0.055 scale model of the Apollo and Saturn-1 configuration is used. The one-third octave band sound pressure levels (S.P.L.) obtained from this test for a zero angle of attack and a Mach number of 0.8 is shown in figure 3, and the magnitude of this S.P.L. is assumed to be constant around the circumference of each band of the grid. The one-third octave S.P.L. at Mach 0.8 from figure 3 is reduced to the spectrum level (db/cps) at the desired center frequency of 89.8 cps by means of the B & K reduction curve shown in figure 10, or by equation 3, and this reduced S.P.L. (db/cps) is then converted to pressure (P_{rms}/cps) at each band station by means of equation 4. The generalized force for a frequency of 89.8 cps is now computed from equation 5 that is

$$F_n = \sum_{\text{All Elements}} \frac{P_{rms}}{cps} \left(\frac{a}{g}\right) \Delta S$$

$$= 2293 \text{ lbs/cps}$$

The detailed computation giving the above generalized force is done in tabular form as shown in table II. It should be noted that in making this computation the fluctuating pressure is assumed to be perfectly in phase with the deflection function of the vibration mode so that there is 100 percent spatial correlation, and this assumption is modified later on to allow for a more realistic spatial correlation factor of 20 percent which is based on experience of a limited number of Mercury flights.

Having calculated the generalized force and generalized mass, the amplification factor that must be applied to the previously calculated "unit" stress of 534 psi (rms) is now obtained from equation 12 using a structural damping coefficient obtained from the vibration test of $\gamma = .006$.

$$\left(\frac{\ddot{U}}{g}\right)_{rms} = \frac{1}{g}\left(\frac{\omega_n}{8\gamma}\right)^{\frac{1}{2}} \frac{F_n}{M_n} = \frac{1}{32.2}\left(\frac{2\pi \times 89.8}{8 \times 0.006}\right)^{\frac{1}{2}} \frac{2293}{571}$$

$$= 13.56$$

The rms fluctuating stress expected inflight assuming 100 percent spatial correlation is therefore

$$\sigma_{rms} = 13.56 \times 534$$

$$= 7240 \text{ psi}$$

To obtain the peak dynamic stress, the rms value is multiplied by a factor of 3 which gives a stress that occurs 0.3 percent of the time.

Measured flight data from the Mercury missions MA-2, MA-3, and MA-4 was reviewed to obtain a factor to account for the fact that the spatial correlation of the fluctuating pressures is much less than 100 percent. A factor of 0.2 was deduced as being a reasonable value.

The final dynamic stress is then

$$\sigma = 534 \text{ (13.56)(0.2)(3.0)} = 4350 \text{ psi}$$

Static stresses occurring simultaneously with these dynamic stresses must be added to obtain a final ring stress.

CONCLUSIONS

The preceding method provides a reasonably fast method of predicting dynamic stresses during flight in complex, non-uniform shell structures using vibration and wind tunnel test data. Several simplifying assumptions are used in the analysis, and these should be borne in mind when applying the method. The spatial correlation factor on full scale vehicles is not well defined and cannot be measured directly.

REFERENCES

1. NASA Working Paper: Shell Modes, Frequencies and Structural Damping of Apollo BP-9, unpublished.

2. Thompson, W. T. and Barton, M. V.: The Response of Mechanical Systems to Random Excitation, Journal of Applied Mechanics, June 1957.

3. Handbook of Noise Control, McGraw-Hill.

TABLE I.- TYPICAL GENERALIZED MASS CALCULATIONS. $f_n = 89.8$ cps

GRID / BAND		1	2	→	35	36	$\sum$
A	W	104.82	16.75		16.75	16.75	
	a/g	0.0	0.0		0.0	0.0	
	$(a/g)^2$	0.0	0.0		0.0	0.0	
	$W(a/g)^2$	0.0	0.0		0.0	0.0	26.48
N	W	8.0	4.18		4.18	4.18	
	a/g	0.4	2.0		3.6	3.0	
	$(a/g)^2$	0.16	4.0		12.96	9.0	
	$W(a/g)^2$	1.28	16.72		54.17	37.62	1320.07
Z	W	24.82	14.45		14.45	14.45	
	a/g	0.0	0.0		0.0	0.0	
	$(a/g)^2$	0.0	0.0		0.0	0.0	
	$W(a/g)^2$	0.0	0.0		0.0	0.0	1.58

$$M_n = \frac{18421.17}{g}$$

TABLE II.- TYPICAL GENERALIZED FORCE. f_n = 89.8 cps

BAND	S ft^2	p_{rms}/cps	$\sum \lvert a/g \rvert$	$S \times p_{rms}/cps \times \sum \lvert a/g \rvert$
A	2.165	4.1	3.4	30.18
B	1.157	3.9	8.6	38.81
$\longrightarrow$				
Y	0.858	1.54	7.6	10.04
Z	1.305	1.54	0.7	1.41
			$\sum$	2293.32

TABLE III. - DAMPING RATIO

Mach Numbers = 0.8 α = 0

Frequency	Damping Ratio, γ
46.5	0.005
61.5	0.004
89.8	0.006
92.8	0.006
137.0	0.004
161.5	0.002
177.4	0.004
226.4	0.004

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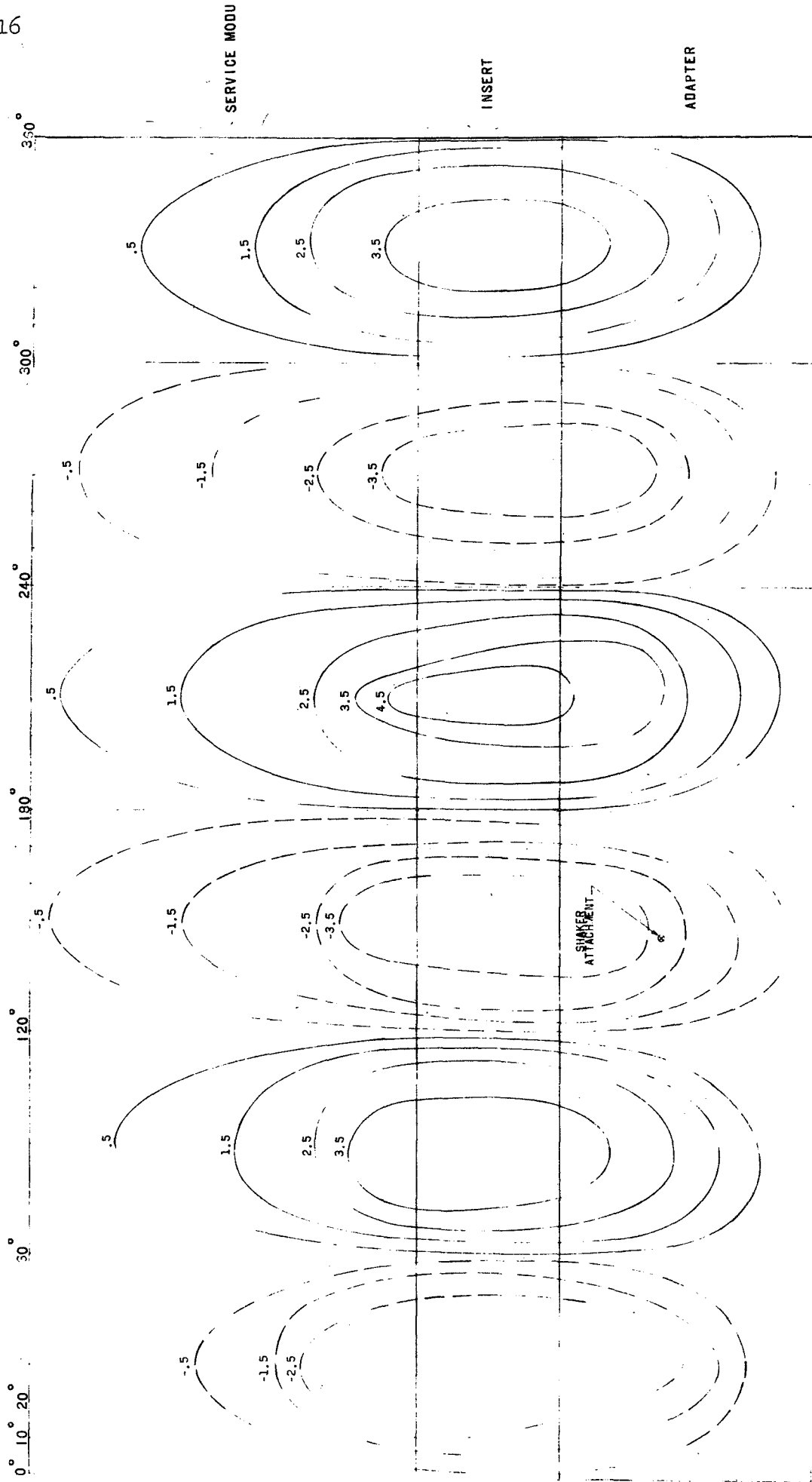
Figure 1.– Contour map of constant acceleration (g's peak) at f = 89.8 cps
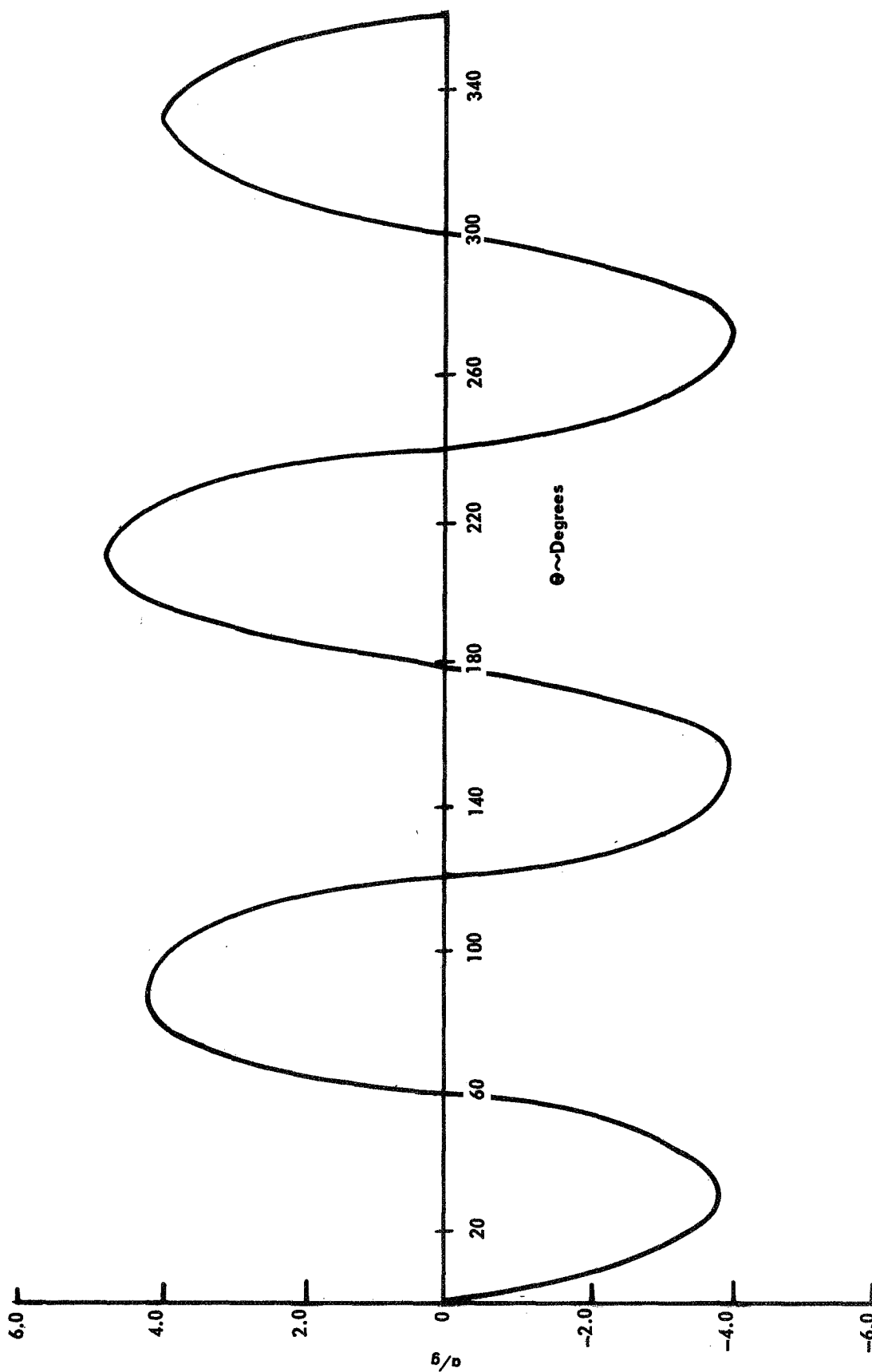
Figure 2.- Typical displacement (a/g) around band N. f_n = 89.8 cps
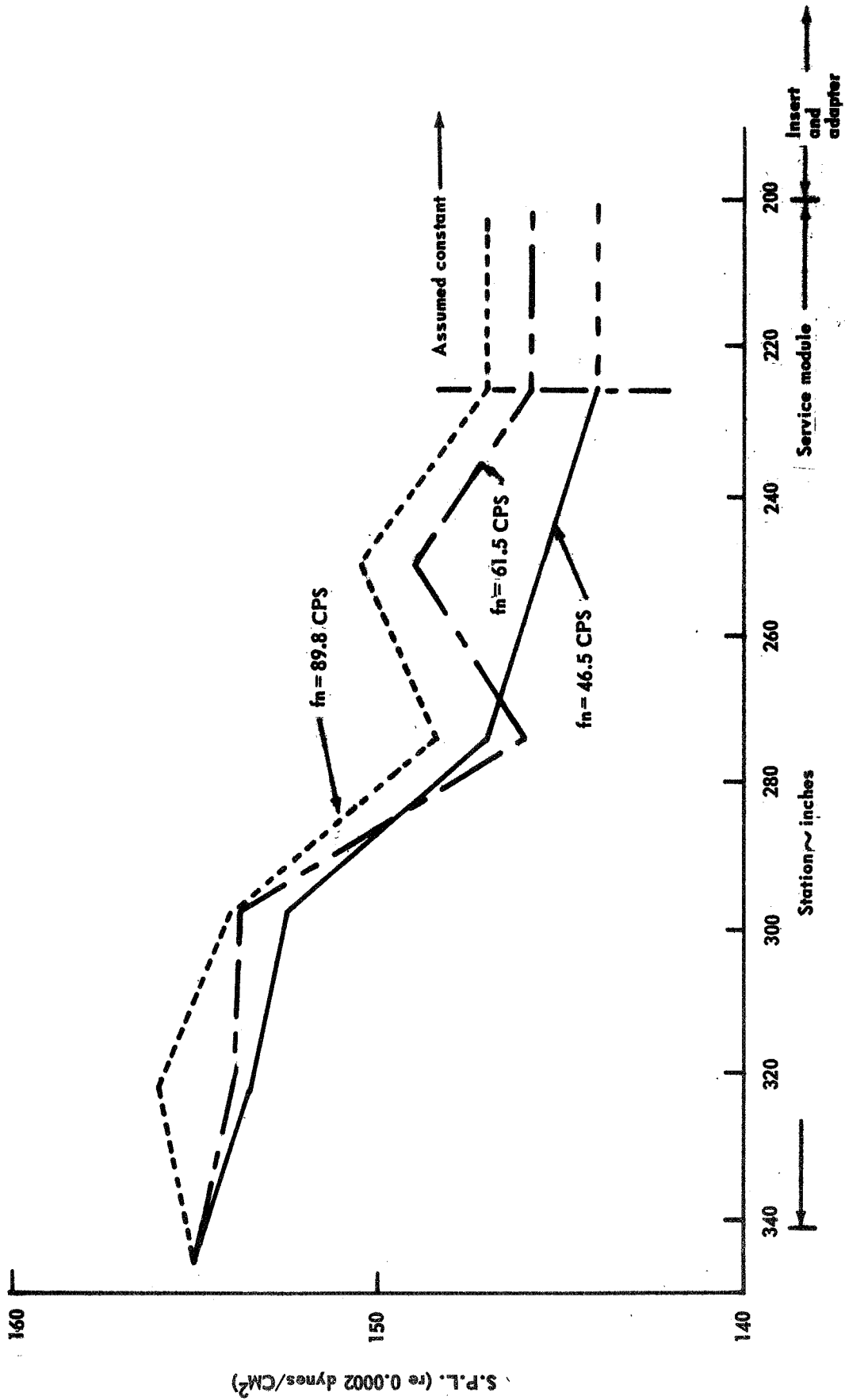
Figure 3.- One-third octave band spectra - Mach number = 0.8 α = 0
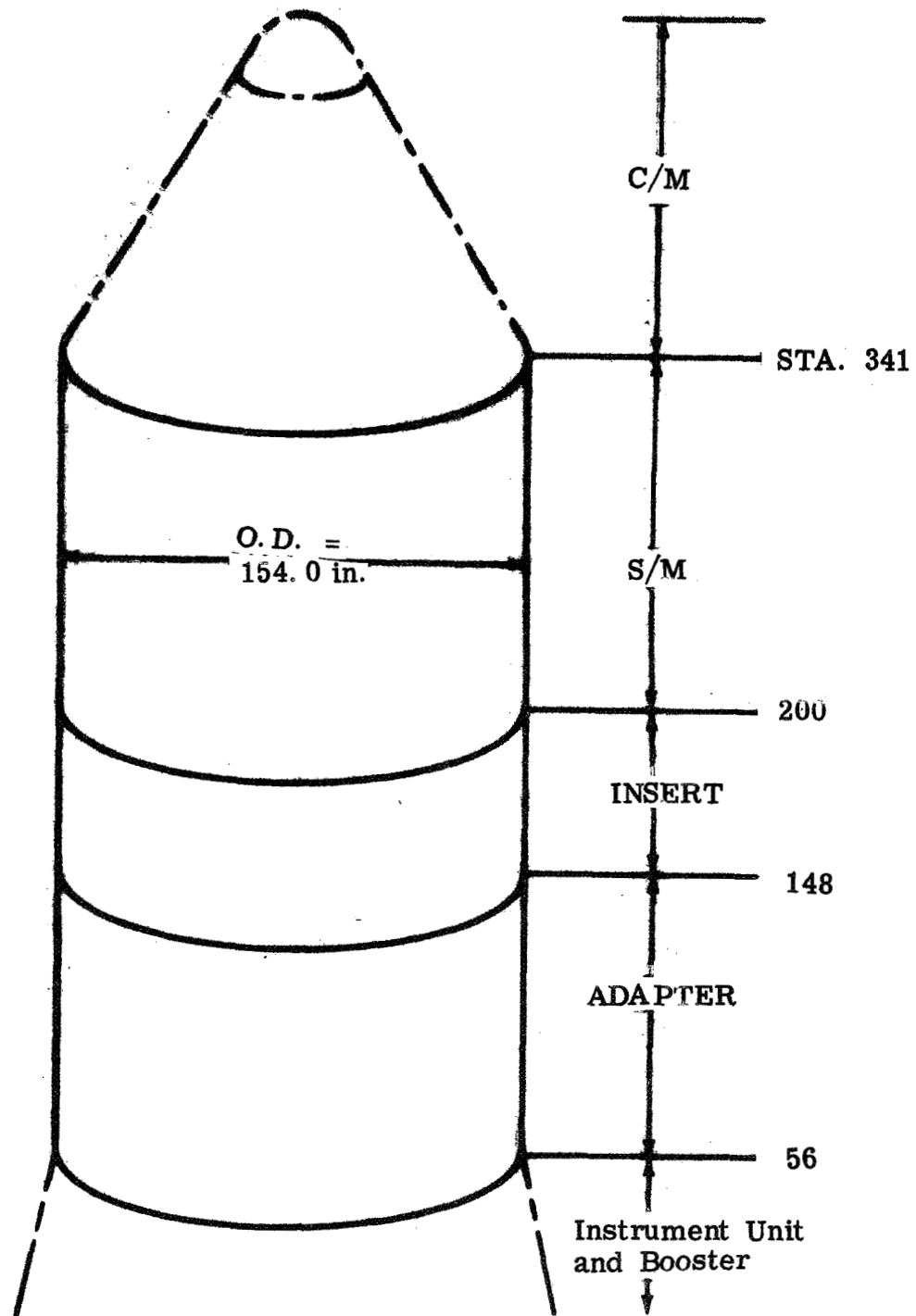
Figure 4.- Boilerplate general arrangement
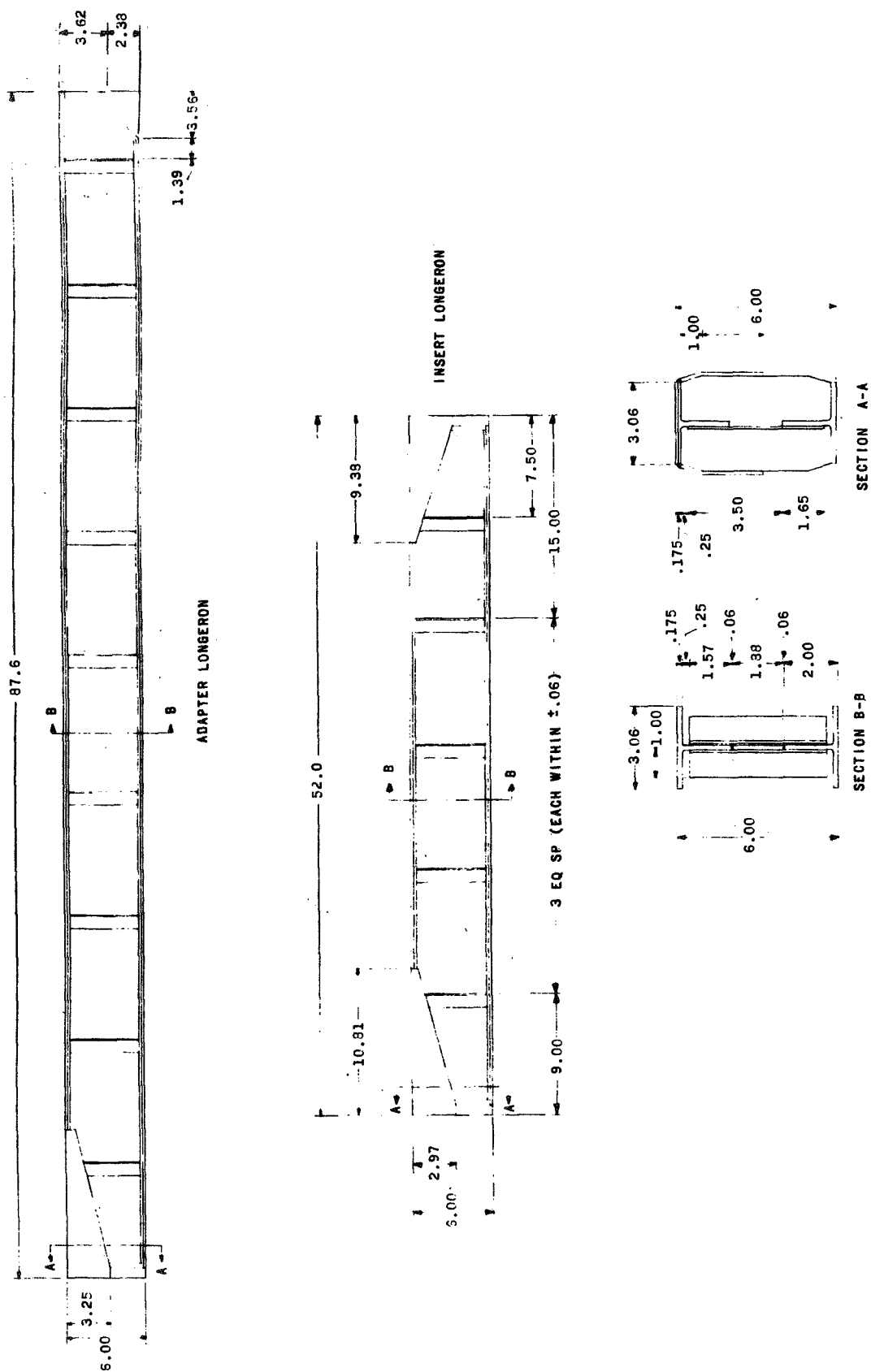
Figure 5.- Structural details of BP-9 insert and adapter longeron

SERVICE MODULE LONGERON
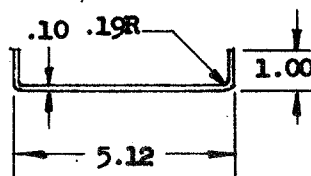
.06 MAX GAP

SECTION A-A

Figure 6.- Structural detail of BP-9 service module longeron
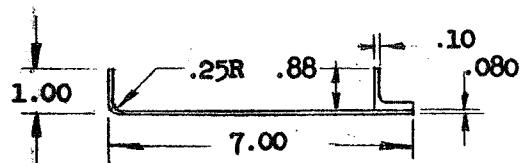
a. Typical insert and adapter ring
   Station 65.3 to station 138.4
   Station 155.4 to station 190.6
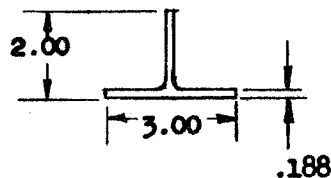
b. Typical stringer for insert and adapter
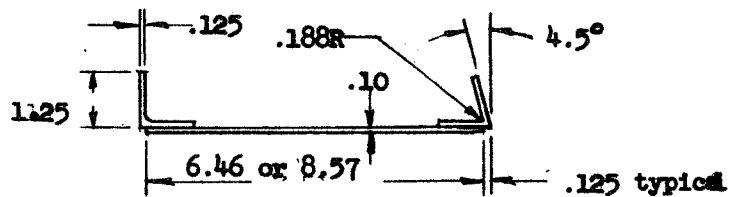
c. Typical service module ring
   Station 212.4 to 237.2

d. Typical service module ring
   Station 249.6 to 311.6

Figure 7.- Typical structural details of BP-9 rings and stringers

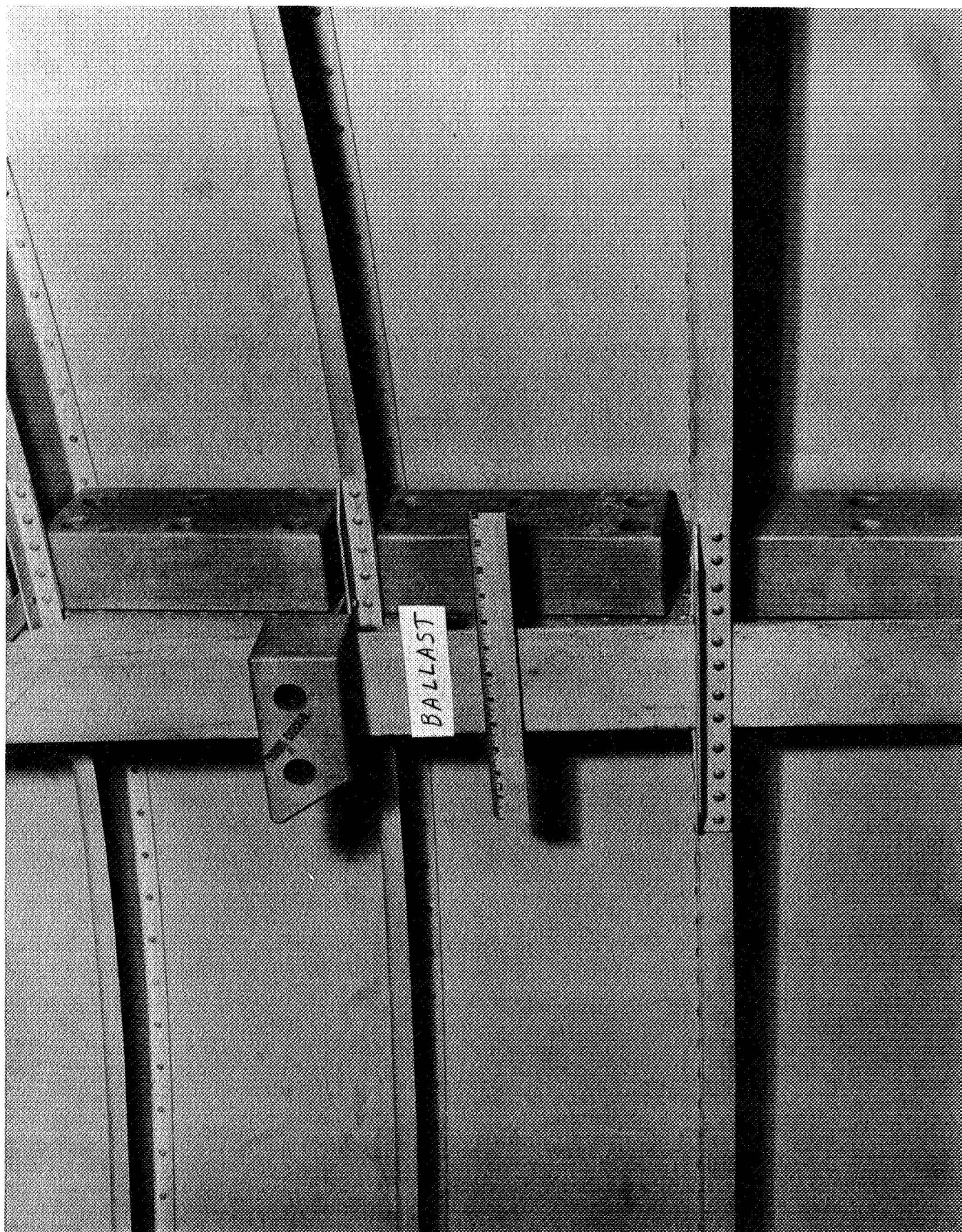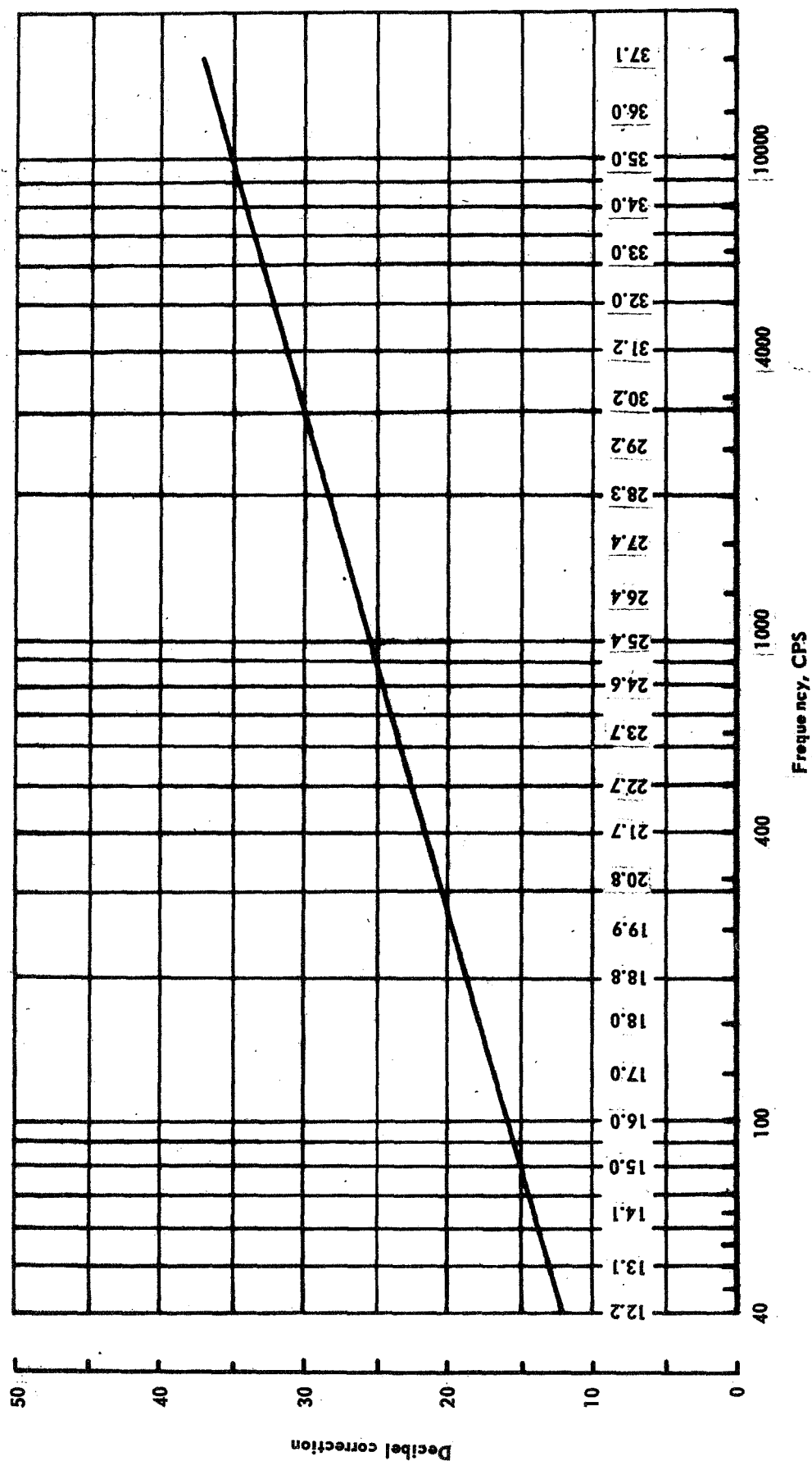Figure 8. - Developed view of interior with grid

Figure 9.- Typical ballast installation in BP-9 service module

db values to be substracted from recorded
B and K data to reduce it to spectrum levels

Figure 10.- Correction curve to reduce one-third octave band to spectrum level